PIGI

Primary Objectives:

Students will:

- Realize the connections between mathematics and the real world.
- Understand the meaning of probability and how it is expressed.
- Comprehend and apply basic concepts of probability.
- Perform simulations to estimate probabilities.
- Make informed decisions about practical situations using probability concepts.
- Move flexibly between concrete and abstract representations of mathematical ideas in order to solve problems, model mathematical ideas, and communicate solution strategies.
- Identify the probability of an event as the number of ways a result can happen over the total number of possible results.

Tips on Modifications for Different Grade Levels:

With younger students this lesson is a great chance to work on strategic thinking skills and mental math skills, identifying numbers, adding, matching sets of numbers, counting objects in a set, working on whether one number is larger or smaller than another number, and practicing basic prediction skills. With K students probability is taught by having students practice figuring out what is "More or less likely." Ex: "Do you think you can roll a 6? Let's look on the chart we made and color coded. 2 3's make 6. What else makes 6? What number are we **most likely** to get?"

With 1st, 2nd, and 3rd grade students need to learn strategic thinking skills and mental math skills, they learn probability by practicing more, less, and equally likely as well as certain, probable, unlikely, and impossible.

In 4th and above students begin to calculate probability specifically, need to learn strategic thinking skills and mental math skills, are introduced to ratios, and convert fractions to decimals, and decimals to percents. According to your students' skill levels they can fill out their probability charts with simple fractions, older students can convert those fractions to decimals.

Examples of Possible Academic Standards to Incorporate:

Kindergarten

- 6.5.1 Sort objects and use one or more attributes to solve problems.
- 6.5.2 Re-sort objects using new attributes.
- 6.5.1 Sort objects into sets and describe how the objects were sorted.
- 6.1.7 Recognize the historical development of mathematics, mathematics in context, and the connections between mathematics and the real world.
- 6.1.8 Use technologies/manipulatives appropriately to develop understanding of mathematical algorithms, to facilitate problem solving, and to create accurate and reliable models of mathematical concepts.
- 6.1.4 Move flexibly between concrete and abstract representations of mathematical ideas in order to solve problems, model mathematical ideas, and communicate solution strategies.
- 6.3.1 Identify, duplicate, and extend simple number patterns and sequential and growing patterns.
- 6.3.1 Use a variety of manipulatives (such as connecting cubes, number cards, shapes) to create patterns.
- 6.3.2 Recognize attributes (such as color, shape, size) and patterns (such as repeated pairs, bilateral symmetry).
- 6.1.1 Use mathematical language, symbols, and definitions while developing mathematical reasoning.
- 6.1.2 Apply and adapt a variety of appropriate strategies to problem solving, including estimation, and reasonableness of the solution.

1st Grade

- 6.5.1 Use various representations to display and compare data.
- 6.5.4 Count and compare collected data.
- 6.1.8 Use technologies/manipulatives appropriately to develop understanding of mathematical algorithms, to facilitate problem solving, and to create accurate and reliable models of mathematical concepts.
- 6.1.7 Recognize the historical development of mathematics, mathematics in context, and the connections between mathematics and the real world.
- 6.1.4 Move flexibly between concrete and abstract representations of mathematical ideas in order to solve problems, model mathematical ideas, and communicate solution strategies.
- 6.1.1 Use mathematical language, symbols, and definitions while developing mathematical reasoning.
- 6.5.1 Represent measurements and discrete data using concrete objects, picture graphs, and bar graphs.

2nd Grade

• 6.5.2 Determine whether an event is likely or unlikely.

- 6.5.1 Use and understand various representations to depict and analyze data measurements.
- 6.5.3 Explain whether a real world event is likely or unlikely.
- 6.5.4 Predict outcomes of events based on data gathered and displayed.
- 6.1.7 Recognize the historical development of mathematics, mathematics in context, and the connections between mathematics and the real world.
- 6.1.8 Use technologies/manipulatives appropriately to develop understanding of mathematical algorithms, to facilitate problem solving, and to create accurate and reliable models of mathematical concepts.
- 6.1.4 Move flexibly between concrete and abstract representations of mathematical ideas in order to solve problems, model mathematical ideas, and communicate solution strategies.
- 6.1.5 Use mathematical ideas and processes in different settings to formulate patterns, analyze graphs, set up and solve problems and interpret solutions.
- 6.1.6 Read and interpret the language of mathematics and use written/oral communication to express mathematical ideas precisely.
- 6.1.1 Use mathematical language, symbols, and definitions while developing mathematical reasoning.
- 6.1.2 Apply and adapt a variety of appropriate strategies to problem solving, including estimation, and reasonableness of the solution.

3rd Grade

- 6.5.1 Organize, display, and analyze data using various representations to solve problems.
- 6.5.3 Compare and interpret different representations of the same data.
- 6.5.4 Solve problems using data from frequency tables, bar graphs, pictographs, or line plots.
- 6.5.3 Make predictions based on various representations of data.
- 6.5.2 Solve problems in which data is represented in tables or graph.
- 6.5.1 Interpret a frequency table, bar graph, pictograph, or line plot.
- 6.3.3 Describe and analyze patterns and relationships in contexts.
- 6.1.4 Analyze problems by identifying relationships, distinguishing relevant from irrelevant information, and observing patterns.
- 6.1.8 Explain and justify answers on the basis of mathematical properties, structures, and relationships.
- 6.1.12 Analyze and evaluate the mathematical thinking and strategies of others.
- 6.1.13 Create and use representations to organize, record, and communicate mathematical ideas.

4th Grade

• 6.5.1 Collect, record, arrange, present, and interpret data using tables and various representations.

- 6.5.2 Use probability to describe chance events.
- 6.5.6 Determine a simple probability.
- 6.5.7 Express a probability pictorially.
- 6.5.1 Depict data using various representations (e.g., tables, pictographs, line graphs, bar graphs).
- 6.5.2 Solve problems using estimation and comparison within a single set of data.
- 6.5.4 List all possible outcomes of a given situation or event.
- 6.1.4 Move flexibly between concrete and abstract representations of mathematical ideas in order to solve problems, model mathematical ideas, and communicate solution strategies.
- 6.1.7 Recognize the historical development of mathematics, mathematics in context, and the connections between mathematics and the real world.
- 6.1.8 Use technologies/manipulatives appropriately to develop understanding of mathematical algorithms, to facilitate problem solving, and to create accurate and reliable models of mathematical concepts.

5th Grade

- 6.5.1 Make, record, display and interpret data and graphs that include whole numbers, decimals, and fractions.
- 6.5.2 Describe the shape and important features of a set of data using the measures of central tendency.
- 6.5.1 Depict data using various representations, including decimal and/or fractional data.
- 6.5.2 Make predictions based on various data representations, including double bar and line graphs
- 6.5.3 Calculate measures of central tendency to analyze data.
- 6.1.7 Recognize the historical development of mathematics, mathematics in context, and the connections between mathematics and the real world.
- 6.1.8 Use technologies/manipulatives appropriately to develop understanding of mathematical algorithms, to facilitate problem solving, and to create accurate and reliable models of mathematical concepts.
- 6.1.5 Use mathematical ideas and processes in different settings to formulate patterns, analyze graphs, set up and solve problems and interpret solutions.
- 6.1.4 Move flexibly between concrete and abstract representations of mathematical ideas in order to solve problems, model mathematical ideas, and communicate solution strategies.

6th Grade

- 6.5.1 Understand the meaning of probability and how it is expressed.
- 6.5.1 Understand that the probability of an event is a number between zero and one that expresses the likelihood of its occurrence.
- 6.5.2 Identify the probability of an event as the ratio of the number of its actual occurrences to the total number of its possible occurrences.

- 6.5.3 Express probabilities in different ways.
- 6.5.4 Understand the difference between probability and odds.
- 6.5.5 Analyze a situation that involves probability of an independent event.
- 6.5.6 Estimate the probability of simple and compound events through experimentation or simulation.
- 6.5.7 Apply procedures to calculate the probability of complimentary events.
- 6.5.1 Determine the theoretical probability of simple and compound events in familiar contexts.
- 6.1.7 Recognize the historical development of mathematics, mathematics in context, and the connections between mathematics and the real world.
- 6.1.8 Use technologies/manipulatives appropriately to develop understanding of mathematical algorithms, to facilitate problem solving, and to create accurate and reliable models of mathematical concepts.
- 6.1.4 Move flexibly between concrete and abstract representations of mathematical ideas in order to solve problems, model mathematical ideas, and communicate solution strategies.
- 6.1.5 Use mathematical ideas and processes in different settings to formulate patterns, analyze graphs, set up and solve problems and interpret solutions.
- 6.1.1 Use mathematical language, symbols, and definitions while developing mathematical reasoning.

7th Grade

- 6.5.2 Select, create, and use appropriate graphical representations of data.
- 6.5.5 Understand and apply basic concepts of probability.
- 6.5.2 Interpret and solve problems using information presented in various visual forms.
- 6.5.4 Use proportional reasoning to make predictions about results of experiments and simulations.
- 6.5.6 Use a tree diagram or organized list to determine all possible outcomes of a simple probability experiment.
- 6.5.1 Interpret and employ various graphs and charts to represent data.
- 6.5.4 Use theoretical probability to make predictions.
- 6.1.7 Recognize the historical development of mathematics, mathematics in context, and the connections between mathematics and the real world.
- 6.1.8 Use technologies/manipulatives appropriately to develop understanding of mathematical algorithms, to facilitate problem solving, and to create accurate and reliable models of mathematical concepts.
- 6.1.4 Move flexibly between concrete and abstract representations of mathematical ideas in order to solve problems, model mathematical ideas, and communicate solution strategies.
- 6.1.5 Use mathematical ideas and processes in different settings to formulate patterns, analyze graphs, set up and solve problems and interpret solutions.
- 6.1.2 Apply and adapt a variety of appropriate strategies to problem solving, including estimation, and reasonableness of the solution.

8th Grade:

- 6.5.2 Compare probabilities of two or more events and recognize when certain events are equally likely.
- 6.5.1 Solve simple problems involving probability and relative frequency.
- 6.5.1 Calculate probabilities of events for simple experiments with equally probable outcomes.
- 6.5.2 Use a variety of methods to compute probabilities for compound events (e.g., multiplication, organized lists, tree diagrams, area models).
- 6.3.13 Represent situations and solve real-world problems using symbolic algebra.
- 6.1.7 Recognize the historical development of mathematics, mathematics in context, and the connections between mathematics and the real world.
- 6.1.8 Use technologies/manipulatives appropriately to develop understanding of mathematical algorithms, to facilitate problem solving, and to create accurate and reliable models of mathematical concepts.
- 6.1.4 Move flexibly between concrete and abstract representations of mathematical ideas in order to solve problems, model mathematical ideas, and communicate solution strategies.
- 6.1.5 Use mathematical ideas and processes in different settings to formulate patterns, analyze graphs, set up and solve problems and interpret solutions.

High School: Algebra

- 2.5.2 Use statistical thinking to draw conclusions and make predictions.
- 2.5.3 Understand basic counting procedures and concepts of probability.
- 2.5.1 Identify patterns or trends in data.
- 2.5.12 Use techniques (Venn Diagrams, tree diagrams, or counting procedures) to identify the possible outcomes of an experiment or sample space and compute the probability of an event.
- 2.5.17 Perform simulations to estimate probabilities.
- 2.5.18 Make informed decisions about practical situations using probability concepts.

High School: Algebra 2

- 3.1.6 Employ reading and writing to recognize the major themes of mathematical processes, the historical development of mathematics, and the connections between mathematics and the real world.
- 3.3.12 Interpret graphs that depict real-world phenomena.
- 3.5.3 Use data and statistical thinking to draw inferences, make predictions, justify conclusions and identify and explain misleading uses of data.
- 3.5.4 Develop an understanding of probability concepts in order to make informed decisions.
- 3.5.13 Apply both theoretical and experimental probability to analyze the likelihood of an event.

Examples of Possible Academic Vocabulary to Incorporate: Kindergarten:

- Addition
- Classify
- Compare
- Difference
- Location
- Minus

1st Grade:

- Data
- Equal to
- Estimate
- Even
- Graph
- Greater than/less than
- Horizontal

2nd Grade:

- Dollar
- Elapsed time/time interval
- Equivalent
- Event
- Extend

3rd Grade:

- Change (money)
- Conclusion
- Conjecture

4th Grade:

- Accuracy
- Chance
- Computation
- Convert

- Number
- Order •
- Pattern
- Penny
- Position
- Sort •
- Measure/measure • ment
- One-half •
- Symbol •
- •
- Interpret •
- Likely/unlikely •
- Multiplication
- Outcome
 - Reflect

- Subtraction
- Sum •
- Value
- Zero
- Unit (standard, non-standard)
- Vertical
- Whole
- Reality
- Whole number
- Rotate
- Set
- Table

- Reasonableness •
- mode) •
- Pattern rules
- Probability
- Proper fraction

tally chart

Factor

•

- Equation •
- Measures of central tendency (mean, median,

Frequency table,

- Part

- Total
- Fraction

- •
- •

Plus • Solve •

- Range
- Relationship

5th Grade:

- Data collection methods
- Justify

6th Grade:

- Compound event
- Dependent events
- Experimental
- probability
- Negative • Odds
- 7th Grade:
 - Property
 - Phenomenon
 - Speed
 - Impact

8th Grade:

- Probability
- Mathematical model
- Monetary
- Deductive & inductive reasoning
- Product

- - Recession •
 - Implied •
 - Inherent •
 - Utilization •
 - Vernacular •
 - Social Norms

- Variable
- Simple event
- Simulation
- Theoretical probability
- Distribution
- Proportional relationships
- Real number properties
- Human Impact
- Mean
- Profit
- Combination

High School:

Instructors, please note that though there are no specific Academic Vocabulary lists for high school students, they will be expected to be familiar with and understand the key mathematics terms and concepts covered within the following lesson. Familiarize yourself with the proper terms for all of the following concepts and make sure that you use them with and explain them to your students.

• Random

Percent

• Power

Sample, sample • data

Remainder

Numerical data

Right

Model

Solution

•

•

•

- Similarity •
- Capitalism
- Productive
- Incentives

- •
- - •
 - Marginal

Simulations

PIGI

Playing PIG! Is a really fun way for students to practice doing addition in their heads. This mental math skill is something a lot of students need help with and a chance to practice, in a fun and memorable way. The most obvious math while playing the game might be the simple addition, but playing Pig! is also a great way to teach students how to think strategically to look ahead and figure out how what just happened, and what might happen next, will affect their chances of winning. And this effort to predict provides a perfect chance to explore probability, a way to measure how likely something is to happen. Probability doesn't tell us what will happen, it just tells us how likely something is to happen.

In this dice rolling game players win by getting the score closest to 100, without going over 100.

Materials for each group of students:

- A pair of Dice
- Paper and Pencils
- A copy of the chart
- Markers, crayons, or colored pencils with at least 4 different colors, 11 different colors is even better.

The object: to be the first to score 100 points without going over. If students have a hard time reaching 100 and get frustrated, change the winning score to 50. (Adjust the goal according to the age of your students and their skills.)

How to play:

- 1. To start, each player rolls a dice. The player with the higher number starts playing first.
- 2. They will roll both dice and add the two numbers together. That's their score.

- 3. Now, they roll again. And add that total to their first score. They add the numbers in their head and keep track of their score out loud. Part of the challenge is doing all of the math in their head. If that is too difficult for students at first allow them to write down their scores and total them, but the goal of the game is for students to be able to do the math in their head. Sometimes it helps if you will relate the numbers to money, \$1.00 = 100. Kids often know how to add up change without realizing they know how to add up numbers too.
- 4. They can roll as many times as they want on a turn, unless they roll a 1. If one of the dice comes up as a 1, they get zero points for that turn, and the dice go to the other player. Even worse, if a 1 comes up on **both** dice, not only does the turn end, but the if two 1s are rolled (snake eyes) the snake eats their score back down to zero and the player's entire game total returns to zero, and it's the other player's turn.
- 5. Each person can decide to stop rolling and pass the dice to the other player at any time. Don't forget—if their score goes **over** 100, they lose! Remind students that if they get close to 100, they might want to let the other player have a turn. After the other player takes a turn, they can decide if they want to roll again.
- 6. The winner is the player whose final score is nearer to 100 (or exactly at 100) without going over by even one point!

After students have had the chance to play the game several times, have a discussion about the strategies they used. You may want to list their ideas and have them test different strategies against each other to try and determine the best way to play.

Exploring Probabilities with PIG!

Understanding the probability that a certain number will come up can help you figure out strategies for winning PIG!

When rolling a single die every number has an equal chance of coming up. You might think that the same is true when you play with two dice, but it isn't. We're going to have students discover this for themselves and discover as well why they are more likely to roll some numbers than others.

Have students fill out the chart showing all the

ways that the numbers on the two dice can be added. Each sum on the chart will show one possible outcome, one of the things that could happen when they roll the dice. How many different possible outcomes are there when they roll two dice?

When you roll two dice, do some numbers come up more often than others? Which numbers come up the most?

Have your students find the answers to these questions using their chart and colored pencils, crayons, or markers.

- Have students use one color to circle all the different ways the numbers on two dice can add up to 7. Ex. You could roll a 1 on Die A and a 6 on Die B, that would equal 7. Or you could roll a 3 on Die A and a 4 on Die B, that also equals 7. How many ways are there to get 7?
- 2. Now, let's go to another number, and another color. Let's try 2. How many ways are there to roll a 2? Have students circle them. They will end up with six circles around numbers that add up to 7 and only one circle around numbers that add up to 2. That means every time they roll the dice, they have six times as many chances to roll a 7 as they do to roll a 2!
- Now have students continue, using different colors to circle all the ways they can roll a 3, 4, 5, 6, 8, 9, 10, 11, and 12. If they don't have 11 different colors, they can use different shapes as well as different colors. Ex. They can put triangles around the

numbers that add up to 12 and circles around those that add up to 10. The point is for them to be able to easily see the different ways they can reach each number. Which number are they most likely to roll?

Experimenting with Probability Pig

To explore probability further divide your students into pairs and have them roll their dice a minimum of 20-30 times, keeping track of their results on a bar graph style chart. This helps your students see what is happening and visually see what numbers are most likely to come up. To have an accurate chart you need to record 200 to 250 rolls. That means if you have 10 pairs of students they only need to roll their dice 20 to 25 times in order for you to have good results, if you have fewer students, they'll need to roll a lot more, or roll

individually, keeping track of their own results.

After everyone has rolled the requisite number of times, ask each student pair how many times they got the number 2. Add all the results and graph the number on the bar graph. Then keep going, repeating this for every number all the way up to 12.

Ask your students what the graph is showing them. It may reveal several things, such as the numbers 2 and 12 don't come up very often and the numbers 6, 7, and 8 get rolled a lot.

If your students compare their charts they filled out earlier and their graphs they may see that the experimental results (their rolls) might not exactly match the predicted results from the probability theory (their charts.) Experimental results show tendencies. Remember, probability isn't a perfect predictor, it just tells us what is likely to happen, not exactly will happen.

Looking at their dice chart can help students see that they are more likely to roll some numbers and less likely to roll others. The different colors, or different shapes creates a visual pattern on their charts, and this makes understanding their dice charts even easier.

Have students count up all of the different possible outcomes with two dice. They will find that when a player rolls two dice there are 36 different combinations possible. Remind students that rolling a 1 on Die A and a 4 on Die B is different than rolling a 4 on Die A and a 1 on Die B, even though you come up with the same answer of 5. This is why the dice on their charts are colored blue and orange. Rolling a 1 on the orange die and a 4 on the blue die is clearly different than rolling a 4 on the orange and a 1 on the blue. Different events have happened, reaching the same result.

While we have 36 possible outcomes, there is still only one way to roll a two and six ways to roll a 7. They are six times more likely to roll a 7 than they are a two.

But, what does it have to do with playing PIG!?

Knowing which numbers are most likely to come up is an advantage when students are playing PIG! Remember, our target score is 100, and we don't want to go over. So if you have a score of 98, are you going to want to roll again? What are your chances of rolling a 2? What if you have 90 points? Should you roll? 10 points and you will win, 11 or 12 and you will lose. To figure out whether to roll again your students can look at their dice chart and count how many ways there are to roll an 11 or

12. There are only three ways to roll and 11 or 12, meaning that their chance of rolling over 10 is only 3/36. So there are 33 ways of rolling a lower number. The might want to roll again, since

their chances are very good that they won't roll over 100, but keep in mind, probability doesn't tell us what will happen, it only tells us what is likely to happen. What if you have 94 points?

Once your student has grasped this concept you can teach him/her how to calculate basic probability. When you flip a coin you have an equal probability of getting heads or tails. So if you say "heads" how likely is it that you are going to win? One way to describe your probability of winning is to say that you have one chance out of two to win. There are two possible events, it will land on heads or it will land on tails. In one of those events you win, in one of them you lose. So your probability of winning is 1 out of 2, or written as a fraction, ½. The number of ways your result (winning) can happen, 1, over the number of possible results (heads or tails), 2.

Probability = <u>The number of ways a result can happen</u> The number of possible results

If your probability was 0, means that that result will never happen, there is no likelihood. If your probability was 1, that would mean that that event would happen no matter what, there is no likelihood of it not happening. So, probability is a number between 0 and 1 that measures how likely it is that something will happen. So in our penny flipping example, a probability of ½ means that that result (you will win) will happen half of the time.

With dice there are a lot more possible outcomes. With one die we have 6 possible outcomes, with two dice we have 36 possible outcomes. It's important to remember that when we are

calculating the a result can number will rolling probability (creating a ratio by comparing the number of ways happen to the number of possible results) that a certain come up when we roll them. So when we were debating again when we had a score of 90, we found our chances of rolling an 11 or 12 were 3.

What's our probability? 3:36 = 3/36 =

1/12. A fraction can be converted into a decimal number. Divide 12 into one and you get 0.08. Then to convert it into a percent multiply .08 by 100. You get 8%.

Now, let's try it the other way. Let's find out our probability of rolling a 10 or smaller. Have students count how many outcomes are possible that result in 10 or a smaller number. There should be 33. So now

what do we do? Create a fraction by putting the number of ways our result (getting 10 or

smaller) can happen over the number of possible results (all of the numbers.) The ratio is 33:36. Which means our fraction is 33/36. So there is our probability. But what does that mean? That means that if we roll the dice 36 times, we are going to roll to roll the number 10 or a smaller number than ten 33 times out of those 36 rolls. We can make the fraction smaller (11/12) and it tells us the same thing, if we roll those dice 12 times, we are likely to get a number 10 or smaller 11 times out of those 12 rolls. That's a pretty good probability! But can we show it as a decimal and as a percent? Yes! Divide 11 by 12. What get? (0.92) Now, multiply your answer by 100. What is the percentage of getting a 10 or smaller? 92% But remember, doesn't tell us what will happen, it tells us what is likely to happen.

So why do people say a coin toss is 50/50? Isn't the probability ½? Let's convert it and find out. Divide 2 into 1. You get what? (.5) Now multiply by 100 to get a percent. (50%) So heads will turn up 50 percent of the time. That's why people say that you have a 50/50 chance.

Have students continue finding probability. What is the probability of rolling 6 or less? What about the probability of rolling snake eyes? Can your students

calculate the probability of pigs flying?

Variations:

The Teacher may wish to play that the first student or Team to 100 is the winner but since it is possible for one student to be SO lucky as to get to 100 without passing the dice, the Teacher may want to add new twists to keep things interesting - first to 100 is the 1st winner and begins SUBTRACTING from 100 back to zero.

As students progress, move on to 12-sided dice. Because the scores add up so much more rapidly and the likelihood of rolling a 1 diminishes greatly, the student might only get 3 rolls per turn, however the same PIG! rules apply.

HOG

In 3rd grade they have been introduced to and SHOULD know their multiplication tables. Then the Teacher may switch to HOG. The rules are the same except that you multiply the numbers on the dice and add it to your score. You may wish to keep students scores on individual note cards.



